

8. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 2 & -3 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 2 & -2 \\ -2 & 4 & 1 & 9 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & 3 & -2 \end{pmatrix}$.

Compute the following matrix sums and matrix products whenever they are defined; if they are not defined, explain why.

(a) A^2 , (b) $A \cdot B$, (c) $B \cdot C$, (d) $B + C$, (e) $A + B$

Math 120: Discrete Mathematics
Second Hour Exam
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Solve all of the following problems. You must show all your work to receive full credit for your solution. Please be concise in your answers. Write clearly and neatly. NO answers should be written on this question sheet.

- The following questions refer to the graph in Figure 1 below.
 - What is the size of the adjacency matrix, A , of the graph. Write down the first, fourth, and fifth rows of A , and the third column of A .
 - Without computing any matrix products, determine the $(3,4)$ -entry of A^3 . Explain your answer.
 - Explain why the diagonal entries of A^2 give the degree of the corresponding vertices of the graph.
 - Does this graph have an Eulerian cycle? If so find one by any means you wish.
- How many labeled graphs are there on twelve vertices; how many of these have exactly seven edges.
- You have a graph with 14 vertices with each vertex having degree three. How many edges does the graph have?
- You have a graph where each vertex has degree at least one. Explain why there must be (at least) two vertices with the same degree.
- Use Dijkstra's algorithm to determine the path of least weight from the vertex labeled A to that labeled H in the weighted graph in Figure 3 below. Show the final labeling of Dijkstra's algorithm.
- Explain why the Euler number of a polyhedron is the same as that of a planar graph.
- Determine the Euler number of a forest. (Hint: Look at the number of trees in the forest.)