11. There are eight students in a room. Define a graph whose vertices are the eight students, with two vertices joined by an edge if those two students have a class in common.
(a) What is the meaning of the degree of a vertex in this graph?
(b) If each student has a class with three other students in the room, how many edges does the graph have?
(c) What does it mean if the graph is a complete graph?

12. (a) What is the third row of the adjacency matrix of the graph sketched on the board?
(b) What is the relationship between the rows and the columns of the adjacency matrix?
(c) Use the breadth–first and depth–first procedures to find two spanning trees in the graph. Describe in words how the two procedures differ.
(d) Does the graph have an Euler circuit? If so find one by any means, otherwise explain why it has none.

12. What is the $(3, 4)$ entry of the 1998'th power of the adjacency matrix of the graph sketched on the board? Explain your answer.

6. (a) How many one–to–one functions are there from a set containing 10 elements to a set containing 6 elements?
(b) How many one–to–one functions are there from a set containing 6 elements to a set containing 10 elements?
(c) How many onto functions are there from a set containing 10 elements to a set containing 6 elements?
(d) How many onto functions are there from a set containing 6 elements to a set containing 10 elements?
Explain your answers.

7. The British author Thomas Hardy set much of his work in a semi–fictional place he called "Wessex". A commentator on Hardy's work wrote the following:

> Many novelists have set their scenes in real places, or have written with some features of a familiar landscape always before them. But Hardy has done something different. Almost every step taken by his characters is taken along real roads or over real heaths; the towns, the villages, even many of the houses, are identifiable . . . Many of the characters, there is little doubt, contain more or less of one real person, more or less of another, with elements drawn purely from imagination or from accumulated layers of experience, which comes to much the same thing. But with the topography, Hardy was rarely satisfied with anything less than a *one–to–one correspondence* between the fictional and the real.

In this context what is the meaning of the phrase in italics. Did the author use the phrase in a mathematically accurate way? Explain your anwer.

8. A positive integer has a 32 bit binary representation in which exactly three places are occupied by a 1.
(a) How many such numbers are there?
(b) How many such numbers are there in which no two of the 1's are next to each other?
(c) How many such numbers are there in which all three 1's appear consecutively?
Explain your answers.

9. (a) Give an example of an onto function. Clearly state the domain, codomain, and range.
(b) Give an example of a one–to–one function. Clearly state the domain, codomain, and range.

10. What is the probability that a randomly chosen list of five letters contains three different consonants and two different vowels? (Count the letter "y" as a vowel.) Explain your answer.

Math 120: Discrete Mathematics
Final Exam
13 May 1998

Solve all of the following problems. You must show all your work to receive full credit for your solution. Please be concise in your answers. Write clearly and neatly. NO answers should be written on this question sheet.

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1. An investor is going to donate a total of 1000 shares of stock to Bradley University. She will first select a six companies from a list of ten companies prepared by her broker and then decide how to divide the stock up among the companies.
(a) In how many ways can she select the six companies?
(b) In how many ways can she divide up the stock among the six companies?
(c) In how many different ways can the donor make her gift to the University?
Explain your answers.

2. Define a function as follows:

$$f(1) = 2 \quad \text{and, for } n > 1\,, \quad f(n) = -7 + f(n-1).$$

(a) Evaluate $f(2)\,,\ f(3)\,,\ f(4)\,,\ f(5)\,.$
(b) Find a simple closed form formula for $f(m)\,.$

3. Show that for $n > 5\,,\ 1! + 2! + 3! + \cdots + (n-1)! + n! \equiv 3 (\mathrm{mod}\, 10)\,.$ Is this also true for $n \leq 5\,?$ Explain your answer.

4. A typical adult human head has about 375 cm^2 of scalp. The number of hairs on a typical adult human scalp is about 400 per $\mathrm{cm}^2\,.$ Must there be two people in the United States with the same number of hairs on their heads? Is the same true of people living in the Chicago metropolitan area? How about of students enrolled in Bradley University? Explain your answers.

5. Show that $\dfrac{1}{1\cdot 2} + \dfrac{1}{2\cdot 3} + \dfrac{1}{3\cdot 4} + \cdots + \dfrac{1}{(n-1)\cdot n} + \dfrac{1}{n\cdot(n+1)} = \dfrac{n}{n+1}\,.$