Math 120
Final Exam
(from a previous semester)

Show all work and mark answers clearly. You may leave factorials and the notation $C(n, k)$ and $P(n, k)$ in your answers. The counting and probability problems are worth 5 points each.

1. A city council is composed of 20 liberals and 18 conservatives. Five members are to be selected randomly as delegates to a convention.

(a) How many delegations are possible?

(b) How many delegations could have 3 liberals and 2 conservatives?

(c) What is the probability that there are no conservatives in the delegation?

2. Consider a 10-bit string of 0's and 1's.

(a) How many strings begin 00 or end 11 (or both)?

(b) How many strings have precisely four 1's?

3. How many ways are there to order 10 hot dogs from 3 types (regular, super, and chili) if at least one hot dog of each type must be chosen?

4. How many distinct arrangements are there of the letters in MATHEMATICS?

5. How many subsets does the set $\{2, 4, 8, 16, \ldots, 128\}$ have?

6. If a six-sided die is rolled 10 times, what is the probability that it will land "ONE" precisely 3 times?

7. A music company executive must decide the order in which to present 6 selections on a compact disk. How many possibilities are there?

8. (15 points) Prove carefully using mathematical induction:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

9. (15 points) For parts $(a)-(c)$, a relation R is given on the set S of all integers. Determine in each case whether R is reflexive, symmetric, or transitive and mark the appropriate box.

	reflexive	symmetric	transitive
(a) $x\,R\,y$ provided $x \le y$	☐	☐	☐
(b) $x\,R\,y$ provided $x-y$ is even	☐	☐	☐
(c) $x\,R\,y$ provided $x^2+y^2=1$	☐	☐	☐

For each of the relations above which is an equivalence relation, do the following:
Describe [1], the equivalence class containing the integer 1.

10. (20 points) Perform each of the following operations in Z_{12} (i.e., perform modulo 12 arithmetic) and write your answer in the form $[r]$ with $0 \le r < 12$.

(a) $[7]+[8]=$

(b) $[3]\cdot[11]=$

(c) $[13]^5=$

(d) Is $-2 \equiv 14 \pmod{12}$?

11. (10 points) Find the coefficient of x^3y^5 in $(2x-y)^8$.

12. (15 points) A sequence $\{s_n\}$ is defined by $s_0=1$, and $s_n=5s_{n-1}+4$ for $n\ge 1$.

(a) Compute s_1, s_2, and s_3.

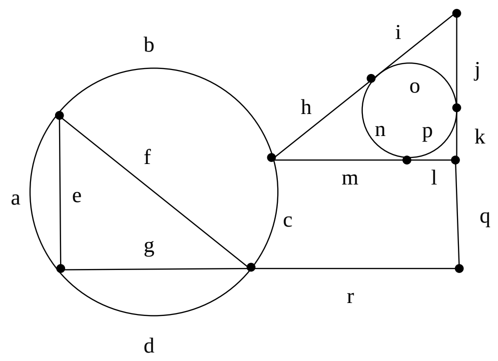
(b) Find an explicit formula for s_n. (Simplify.)

13. (20 points)**Generating Functions.**
(a) Let a_r be the number of ways of taking r bagels from a basket containing 3 plain, 2 onion, and 5 blueberry bagels. Give a generating function for the sequence $\{a_r\}$. (Do not expand your answer.)

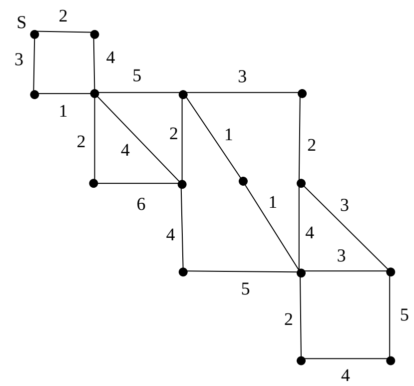
(b) Give a formula for s_n if $S=\dfrac{1}{1-x}+\dfrac{1}{1+2x}$ is a generating function for the sequence $\{s_n\}$.

14. (10 points) Suppose a tree has six vertices of degree 1, eight vertices of degree 2, four vertices of degree 3, and no vertices of degree greater than 4. Explain why the tree can have no vertices of degree 4.

15. (15 points) The graph below has either an Euler circuit or an Euler path. Determine which and justify your answer by actually construcing a circuit or path.



16. (15 points) Find a minimal spanning tree in the graph below and give the weight of the minimal spanning tree found. (Start your algorithm at the vertex S. Shade clearly the edges which are to be included.)



17. (15 points) Let $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

(a) Construct a graph G with adjacency matrix A.

(b) What does the $(2,3)$-entry of the matrix A^3 represent? Use either your graph in part (a) or matrix multiplication to compute this entry in A^3.