5. (a) In a few words, and with an appropriately labeled sketch, explain what Simpson's rule does to achieve an approximation to an integral. (Do not give the formula for Simpson's Rule, rather explain what is going on geometrically.)
(b) Give a condition on the function $f(x)$ that guarantees that the trapezoidal approximation gives an overestimate to $\int_a^b f(x)\,dx$. Explain your answer.

6. The table to the right gives values for the functions $f(x)$ and $g(x)$ at selected points.
(a) Evaluate $\frac{d}{dx} f(x)\cdot g(x)$ and $\frac{d}{dx}\frac{f(x)}{g(x)}$ at the point $x = 1$.
(b) Evaluate $\frac{d}{dx} f(g(x))$ and $\frac{d}{dx} g(f(x))$ at the point $x = 0$.

x	-1	0	1	2	3
$f(x)$	3	3	1	0	1
$g(x)$	1	2	2.5	3	4
$f'(x)$	-3	-2	-1.5	-1	1
$g'(x)$	2	3	2	2.5	3

Math 121: Calculus I

Second Exam

24 October 1997

Solve all of the following problems. You must show all your work to receive any credit. Please DO NOT simplify any of your answers. Be complete but concise in your answers.

1. Compute the following. DO NOT simplify your answers.

(a) $\dfrac{d}{dx}x^3 + 17x^2 - \dfrac{1}{x^2} + \sqrt{x}$

(b) $\dfrac{d}{dx}\sin^3(x^2+2)$

(c) $\dfrac{d}{dx}\dfrac{x}{\ln(\cos(x^3))}$

(d) $\dfrac{d}{dx}\sqrt{3 + x\cos^2(x)}$

2. Find the equation of the tangent line to the graph of the equation $\ln(x+y) = y^2$ at the point $x = 1$, $y = 0$.

3. Let $f(x) = \displaystyle\int_1^x \frac{\sin(t)}{t}\,dt$.

(a) Estimate $f(2)$ using a left–hand endpoint approximation with four subintervals.

(b) Find the exact value of $f(1)$, $f'(1)$, $f''(1)$.

4. (a) Indicate on the graph on the left what $\int_2^6 f(x)\,dx$ is measuring.

(b) Indicate on the graph on the right what the midpoint approximation to $\int_4^8 f(x)\,dx$ with 2 subintervals is measuring.