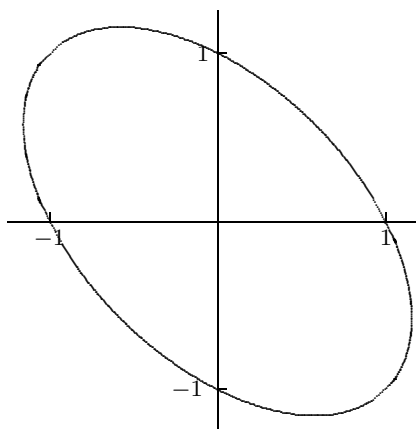


(20) 5. The graph of the curve with equation

$$1 = x^2 + xy + y^2$$

is given on the right.



(a) On the graph, indicate the approximate location of the points at which the derivative, $\frac{dy}{dx}$, is zero.

(b) Find $\frac{dy}{dx}$.

(c) What is the equation of the tangent line at the point $(0, 1)$?

(d) Find the exact x -coordinate of the points where the tangent line is horizontal.

(15) 6. A baseball diamond is a square 90 feet on a side. A ball is batted toward third base, and moves at a constant speed of 100 feet per second. How fast is the distance from first base to the ball changing when the ball is half way to third base?

(10) 7. As a spherical mothball evaporates, its volume decreases at a rate proportional to its surface area. Show that the radius of the mothball decreases at a constant rate.

(Note: The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, its surface area is $4\pi r^2$.)

Math 121: Calculus I

Second Exam

26 October 1998

Solve all of the following problems. You must show all your work in order to receive any credit for your answers. The point value of each problem is given before each problem. Please do not simplify your results.

(20) 1. Find the derivative; do not simplify your answers.

(a) $x^3(x^5 + 4x^3)^6$

(b) $\ln(1 + \sqrt{x})$

(c) $\arctan(\sin(x^3))$

(d) $\frac{4 \cos x}{2 + \sin x}$

(10) 2. Find the exact value of the following limits.

(a) $\lim_{x \rightarrow \pi} \frac{2^{\sin x} - 1}{x - \pi}$

(b) $\lim_{x \rightarrow 0^+} x \ln x$

(15) 3. The number of people, N , who have heard a rumor spread by mass media is often modeled by the function

$$N = a(1 - e^{-kt}),$$

where t is in days since the rumor started spreading, a and k are constants.

(a) Determine $\lim_{t \rightarrow \infty} N$. In common terms, what is this limit measuring?

(b) Assuming 200,000 people eventually hear the rumor and 10% of them hear it on the first day, what are the values of a and k .

(c) Suppose $N = 100,000(1 - e^{-.105t})$. At what rate is the rumor spreading at the start of the third day?

(10) 4. The graph of the hyperbolic sine function,

$f(x) = \sinh(x)$, is given on the right.

(a) Explain why $f(x)$ is invertible.

(b) Find $\frac{d}{dx} \sinh^{-1}(x)$, where $\sinh^{-1}(x)$ stands for the inverse of the hyperbolic sine function. Do not simplify your result.

