Math 122 EXAM #2

Name _____

I.D. # _____

1. (20 pts) The density function for the duration of telephone calls within a certain city is

$$p(x) = \begin{cases} \frac{1}{2}e^{-\frac{1}{2}x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the duration in minutes of a randamly selected call.

(a) What percentage of calls last between 1 and 2 minutes?

(b) What percentage of calls last 1 minute or less?

(c) What percentage of calls last 3 minutes or more?

(d) Find the cumulative function.

(e) Find the mean duration of telephone calls.

2. (24 pts) Solve the differential equation with the initial condition. Write y as a function of x and determine the constant.

(a) $\frac{dy}{dx} = -2y, \quad y = 5$ when $x = 0$

(b) $\frac{dy}{dx} = \left(\frac{x}{y}\right)^2, \quad y = 2$ when $x = 0$

(c) $\frac{dy}{dx} = 2y(3 - y), \quad y = 1$ when $x = 0$

3. (10 pts) Sketch the slope field of the equation $\frac{dy}{dx} = -2y$ and sketch the solution pass through $y(0) = 5$.

4. (10 pts) Use Euler's method for the equation $\frac{dy}{dx} = \left(\frac{x}{y}\right)^2$, $y = 2$ when $x = 0$ to approximate $y(.2)$ by using $\Delta x = .1$. $Show\ your\ work$ and keep five decimals!

5. (16 pts) When a murder is committed, the body, originally at 37°C, cools according to Newton's Law of Cooling, i.e., the body cools at a rate proportional to the difference between the temperature of the body and that of its surrounding. A dead body was found at 5am at a temperature of 32°C and the surrounding air was a constant of 15°C. The body's temperature was measured again at 6am which was 31°C. When was the murder committed?

6. (20 pts) The acceleration of a moving object is given by

$$\frac{d^2x}{dt^2} = x(\frac{dx}{dt} - 1)$$

where $x(t)$ is the position at time t.
(a) Set up a system of first order differential equations for the position x and velocity v.

(b) Find $\frac{dv}{dx}$

(c) The slope field in the vx-phase plane is shown below. Sketch the trajectory for the initial conditions $x(0) = 3,\ v(0) = -2$. Indicate the direction with the arrow. $Show\ how\ you\ get\ that\ direction$!

(d) Use Euler's method to approximate $x(.2)$ by using Δt $= .1$. $Show\ your\ work$!