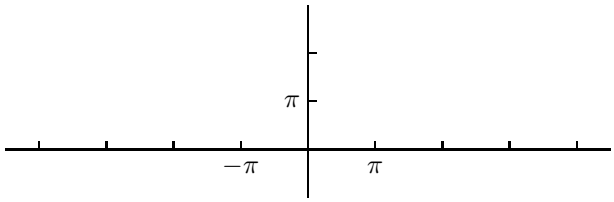
MATH 122
Test IV Name_____
(20 pts each)

1. a) Use a quadratic Taylor polynomial to give a good approximation of $\tan(35^\circ)$. Tell what function f you are using and what you have chosen for a. (Radians!!)

$f(x) =$_____ ; $a =$_____

b) Give a bound on the error in the approximation in (a). Tell how you are finding your bound.

2. Let $f(x) = x + \pi$ on the interval $[-\pi, \pi]$ and extend f to be a periodic function on the whole line.



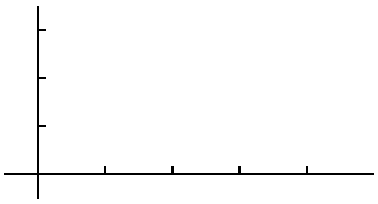
a) Sketch a graph of at least 3 periods of f.

b) Find $F_3(x)$, the third Fourier "polynomial" for f.

c) Graph F_2 on the same graph as f.

3. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{e^x - 1}{2y}$.

a) (The world's most sparse slope field.) Draw three (and 3 only) small tangent lines on the graph below. Draw at the points: $(0, 2)$, $(1, 1)$, $(3, 1)$.



b) Suppose that $y(x)$ is a solution to this differential equation. Tell the slope of a solution passing through $(-1, 2)$.

c) Find the solution of the differential equation satisfying $y(0) = 2$.

FREEBIE: $y = \sqrt{e^x - x + C}$

d) Let's see how accurate Euler's method is with the initial conditions given in (c).

i) Use Euler to approximate $y(2)$, taking 10 steps. (Write the first two steps out so that it is clear that I know that you know Euler's method. Then let your calculator give y_{10}.)

$y_1 =$
$y_2 =$
$y_{10} =$

ii) Compare with the actual value from your solution in (c).

iii) Approximate $y(2)$ again, this time using 20 steps. Again compare with the actual solution.

$y_{20} =$

4. Solve the differential equations.

a) $\dfrac{dy}{dx} = \tan(x)\sqrt{y+1}$

b) $\dfrac{dy}{dx} + \dfrac{4}{x}\,y = x^2 \;\; ; \;\; y(1) = 2$

5. Morphine is often used as a pain-relieving drug. The half-life of morphine in the body is 2 hours. Assume the rate at which morphine is eliminated is proportional to the amount present. Assume an initial dose of 2.5 mg is given.

a) Write a differential equation for the quantity Q of morphine in the blood at time t.

b) Solve the differential equation from (a).

c) Rather than being administered one time only, pain medication is often administered continuously via an intravenous tube or a patch. Suppose that morphine is administered to a patient intravenously at a rate of 2.5 mg per hour. The rate of change of the quantity of morphine in the blood is the "rate in" minus "the rate out." Write a modification of the differential equation in (a) that will describe this new situation. (Need not solve the D.E.)

d) (Continuation of (c).) If we wait long enough, the amount of morphine is the blood will approach an "equilibrium solution" where there is no change in the amount Q. Find this equilibrium amount.