

T___F___ 8. The integration-by-parts formula comes from the product rule for differentiation.

T___F___ 9. If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$, then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 1$.

T___F___ 10. If $p(x)$ is any polynomial, then $\lim_{x \rightarrow \infty} \frac{p(x)}{e^x} = 0$.

T___F___ 11. $\int_0^1 \frac{1}{x^{1.01}} dx$ converges.

T___F___ 12. $\int_1^{\infty} \frac{1}{x^{1.01}} dx$ converges.

T___F___ 13. L'Hôpital's Rule gives no information about $\lim_{x \rightarrow 0} \frac{\cos x}{x}$.

T___F___ 14. $x^{5/2}$ has no Taylor series about $a = 0$.

T___F___ 15. I survived my first year at Bradley University!

x. Suppose you have two differentiable functions $f(x)$ and $g(x)$ which are equal at some point a , i.e. $f(a) = g(a)$ and furthermore suppose $f'(x) = g'(x)$. What conclusion can you draw about $f(x)$ and $g(x)$? Explain your answer.

x. What is the error in estimating the value of the function $f(x) = x^{1998} + x^{1997} + \dots + x^2 + x + 1$ by means of the Taylor polynomial of degree 1999 about the point $a = 2.5$? Explain your answer.

In each of the following, explain whether the statement is true or false and give a brief reason for your answer.

T___F___ 1. If we know that a series converges, we can usually estimate the exact sum of the series.

T___F___ 2. If the series of positive terms $\sum_{k=1}^{\infty} a_k$ converges, then the series $\sum_{k=1}^{\infty} (-1)^k a_k$ converges.

T___F___ 3. If the sequence $\{a_k\}_1^{\infty}$ converges to some finite value L , then $\lim_{k \rightarrow \infty} \frac{a_k}{k} = 0$.

T___F___ 4. The function $f(x)$ and the Taylor polynomial of $f(x)$ of degree n about the point a both have the same $(n + 1)$ 'st derivative at a .

T___F___ 5. The magnitude of the error in approximating $\int_a^b f(x) dx$ by means of the trapezoidal rule with 20 subdivisions is approximately 1/8 the magnitude of the error when approximating it with 10 subdivisions.

T___F___ 6. If the power series $\sum_{k=1}^{\infty} a_k(x - 2)^k$ converges at $x = 4$ then it converges at $x = 0$.

T___F___ 7. Simpson's Rule is exact (i.e. has zero error) when used to estimate the integral of any cubic polynomial.

Math 122: Calculus II
Final Exam, In-Class
7 May, 1998

Solve all of the following problems. You must show all your work to receive full credit for your solution. Unless otherwise stated, you may use your calculator only to check your work, it may not be your only method of solution. Please be concise in your answers. Write clearly and neatly. NO answers should be written on this question sheet.

x. Evaluate the following integrals.

(a) $\int \frac{\ln(1+x)}{1+x} dx$ (b) $\int \frac{4+x}{\sqrt{x}} dx$ (c) $\int (x-1)e^{-x} dx$ (d) $\int \frac{dx}{\sqrt{4-x^2}}$

x. A tent has a base in the shape of a circle of radius 1 meter. Each cross section of the base is in the shape of an equilateral triangle. (See the figure on the board.) What is the volume of the tent?

x. A student has a slow leak in her bike tire. The pressure in the tire decreases at a rate proportional to the difference between the atmospheric pressure (15 lbs.) and the tire pressure. On Monday at 6:00p.m. she pumped up her tire to 85 lbs. By 6:00p.m. on Tuesday, it was down to 75 lbs. How much longer can she wait to pump up the tire if she doesn't want the tire pressure to get below 40 lbs? (You can leave your answer in terms of days since Monday at 6:00p.m., if you wish.)

x. (a) Find the radius of convergence of the power series $\sum_{k=1}^{\infty} (-1)^k \frac{x^k}{k \cdot 3^k}$.

(b) For which of the endpoints of the radius of convergence, if any, does the series converge?

x. Given the differential equation $\frac{dy}{dx} = y(1-x-y)$ and the initial value $y(-1) = 1$, estimate the value of $y(.5)$ using three steps of Euler's method.