MATH 122 Final Exam, Part 1, May 1997

You will have up to 30 minutes to complete this part of the exam. No calculators are permitted on this part. Point values are shown.

1.(10) Find the area beneath $f(x) = xe^x$, between $x = 0$ and $x = 1$, and above the x-axis.

2.(10) a. State the Mean Value Theorem

b. Sketch a graph that shows what it is telling us.

c. Explain in two or three sentences, in your own words, what it says.

3.(10) Integrate: $\int \frac{(x-1)^3}{x^2} dx$

4.(10) Approximate $\int_0^{2\pi} \frac{\sin x}{x} dx$ using the Midpoint Rule with three intervals.

5.(10) Use the definition of Taylor polynomial to compute $P_3(x)$ for the function $\sqrt{x}$ expanded about $a = 4$.

6.(10) Find the general solution to $y'' - 3y' + 2y = 0$

7.(10) Find the sum of the series: $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \cdots$

MATH 122 Final Exam, Part 2, May 1997

Calculators are permitted on this portion of the exam. Write any formula that you use your machine to evaluate. A correct solution with no work shown may receive zero points.

1.(10) a. Write the Taylor series for $f(x) = e^x$.

b. Find a series expansion for $g(x) = e^{-x^2}$.

c. On what interval does the series in (b) converge to e^{-x^2}?

d. Find a series expansion for $h(x) = -2xe^{-x^2}$.

e. What is the interval of convergence for the series in (d)? Why?

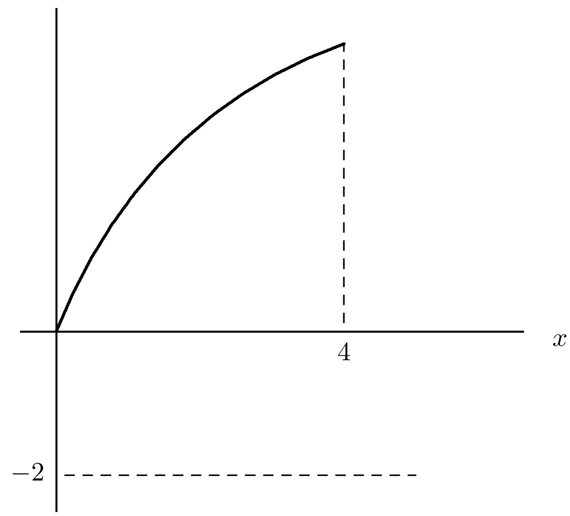
f. Find a series representation for $\int e^{-x^2} dx$

2.(10) Show all of your work to compute $\int x^2 \sin(\pi x) dx$

3.(10) a. Why is $\int_{1/3}^{1} \frac{1}{\sqrt{3x-1}} dx$ an improper integral?

b. Determine if the integral in (a) converges or diverges. Show your work.

4.(10) SET UP a definite integral to compute the volume of a solid of revolution if $\ln(x+1)$, $0 \le x \le 4$ is rotated about the horizontal line $y = -2$.



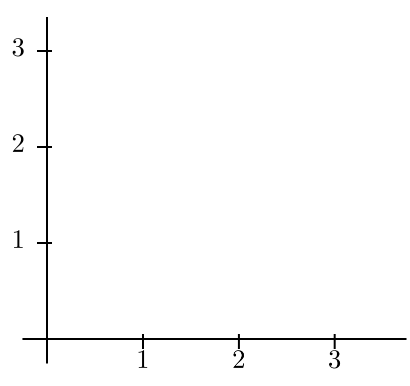
5.(10) WORK EITHER (a) OR (b)

a. How much work does it take to pump a liquid with density w lb/ft^3 from a full upright, right circular cylindrical tank of radius 5 ft and height 10 feet to a level 4 feet above the top of the tank?

b. A tank contains 10000 gallons of fresh water. Water that has a salt concentration of 0.5 lb/gal is flowing into the tank at a rate of 10 gal/min. The salt water is mixed and the tank is draining water also at 10 gal/min. How long will it take for the water in the pool have a concentration of 0.25 lb/gal?

6.(10) Consider the system $\frac{dy}{dt} = y\,(3 - y - x)\,,\ \frac{dx}{dt} = x\,(2 - x - 2y)$.

a. Locate all nullclines and equilibrium points and sketch them in the phase plane of this system.



For (b) assume that $x \geq 0$, $y \geq 0$.

b. Sketch the three trajectories that start from (1, 3), $(1, 1)$ and (.5, .5).

7.(10) a. Find the quadratic Taylor polynomial $P_2(x)$ for the function $f(x) = e^{-2x}$ expanded about $a = 0$.

b. Give an upper bound for the error that comes from using the Taylor polynomial $P_2(x)$ to approximate e^{-2x} at $x = 0.3$.

c. Use your calculator to find the actual error.

8.(10) Consider the differential equation $dy/dx = xy + y$ and initial condition $y(0) = 1$.

a. What is the Euler's approximation to $y(1)$ found using a step size of 0.1? Show the first two steps explicitly. After that, you may simply use your calculator.

b. Find the exact value of $y(1)$ by solving the differential equation.

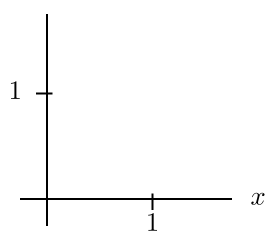
9.(10) (a) Using two subdivisions, find the left, right and trapezoid approximations to $\int_0^1 (1 - e^{-x})\,dx$.

left:

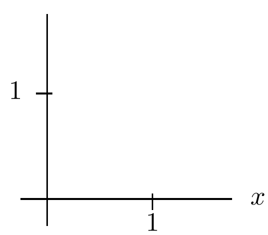
right:

trapezoidal:

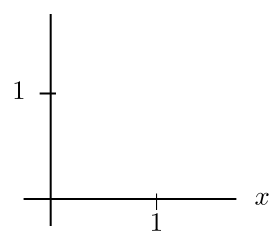
(b) Draw 3 sketches showing what each approximation represents.



Left



Right



Trapezoidal

Given only the information in (a), what is your best estimate for the value of this integral? Explain your answer.

10.(2@) True/False Questions. If your answer is "False," explain why the statement is false.

_____ a. If $\displaystyle\int_0^\infty f(x)\,dx$ converges and if $g(x) > f(x)$, then $\displaystyle\int_0^\infty g(x)\,dx$ diverges.

Reason:

_____ b. Integration by parts cannot be used if the integrand is a quotient.

Reason:

_____ c. The Riemann sum $\sum_{k=1}^{n} \pi\, z_k \sin(z_k)\Delta z$ on the interval $1 \le z \le 5$ may be approximated by $\pi \int_1^5 z \sin z \, dz$.

Reason:

_____ d. The arc length of $\dfrac{1}{\sin x}$ from $\frac{\pi}{4}$ to $\frac{3\pi}{4}$ equals $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sqrt{1 + \left(\frac{1}{\cos x}\right)^2}\, dx$.

Reason:

_____ e. The series $1 + x + x^2 + x^3 + \cdots$ converges for all x in the interval $(-1, 1)$.

Reason: