

Math 122 FINAL

Name _____

I.D. # _____

1. (35 pts) Find the following integrals and state the substitution you use if you use substitution method, state the function U and V (or f and g) if you use the integration by parts, state the formula number if you use the table of integrals.

(a) $\int \frac{2}{3+4x} dx$

(b) $\int \cos x (3 + \sin x)^{99} dx$

(c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(d) $\int \tan^{-1} x dx$

(e) $\int \frac{1}{\sin^3 x} dx$

2. (21 pts) Determine if the following improper integrals converge or diverge. Explain your reasoning.

(a) $\int_1^{\infty} \frac{\sin^4 x + x^2}{\sin^2 x + x^4} dx$

(b) $\int_1^2 \frac{(x+1)^2}{(x-1)^2} dx$

(c) $\int_0^{\infty} \frac{1}{x^2 + \sqrt{x}} dx$

3. (10 pts) Find the volume of the solid obtained by rotating the region bounded by $f(x) = \sin x$, $y = 0$, $x = 0$, and $x = \pi$ about x -axis.

4. (10 pts) A water tank is in the form of a right circular cylinder with height 20 ft and radius 6 ft. If the tank is half full of water, find the work required to pump all of it over the top rim. (Note that 1 cubic foot of water weighs 62.4 lb.)

5. (20 pts) Solve the differential equation with the initial condition. Write y as a function of x and determine the constant.

(a) $\frac{dy}{dx} = x^2 e^y$, $y = 0$ when $x = 0$

(b) $\frac{dy}{dx} = \sqrt{4 - y^2}$, $y = 1$ when $x = \frac{\pi}{6}$

6. (15 pts) Match the four slope fields with four of the differential equations (One equation does not match!) No reason are required.

7. (15 pts) Suppose that at 1:00 pm one summer afternoon, there is a power failure at your house, and your air conditioning does not work without electricity. When the power goes out it is 70°F in your house. At 5:00 pm, it is 76°F in the house. Assume that the temperature, T , in your house changes according to Newton's Law of Heating, i.e., the house warms up at a rate proportional to the difference between temperature in the house and that outside. Assume the temperature outside is 90°F .

- (a) Write the differential equation satisfied by T .
- (b) Solve the differential equation.
- (c) Find the temperature in the house at 10:00 pm.

8. (24 pts)

(a) Write down the Taylor series for e^x at $x = 0$.

(b) Use part (a) to write down the Taylor series for x^2e^x at $x = 0$.

(c) Use part (b) to find $f^{(10)}(0)$ for $f(x) = x^2e^x$.

9. (10 pts) Write down the Taylor series for $\sqrt{1-x^2}$ at $x = 0$.

10. (40 pts)

(a) Write down the Taylor series for $\cos x$ at $x = 0$.

(b) Use part (a) to find the value of the limit $\lim_{x \rightarrow 0} \frac{1-\cos x}{x^2}$

(c) Use part (a) to write down the Taylor series for $x^2 \cos \sqrt{x}$ at $x = 0$.

(d) To what number does the series

$2^2 - \frac{2^3}{2!} + \frac{2^4}{4!} - \frac{2^5}{6!} + \frac{2^6}{8!} - \frac{2^7}{10!} + \dots$
converge?

