MTH207 Exam 1. Fall 98. Be Neat. Show work.

Part I. You may only use your brain, pencil and paper on this portion. When you complete this portion turn it in to receive Part II on which you may use your favorite computational crutch. Each problem (not part) in this portion of the exam is worth the same number of points. Each problem is worth 10 points. Be neat. Show work.

1. Perform the indicated operations or explain why it is not possible to do them.

a) $3\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 3 \\ -1 & 2 & -3 \end{bmatrix}^T \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + (-2)\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix} =$

b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$

2. Find the solution set to the following system or explain why it has none.

$$\begin{bmatrix} -2 & 0 & 6 & 0 \\ 1 & 1 & -3 & 2 \\ 0 & 2 & 0 & 4 \\ 1 & 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0.$$

3. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}$. Find A^{-1}.

4. Let $A \in F_{m,n}$ and $B \in F_{n,p}$ Let $A = (a_{ij})$ and $B = (b_{jk})$. Write down the definition of the matrix product AB using this "shorthand" notation.

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Roughly speaking, each problem is worth 10 points, unless indicated otherwise.

1) Write down the augmented matrix for each of the systems below then use it to decide if the system has following has a unique solution, infinitely many solutions, or no solutions. Explain your conclusions in terms of the reduced row echelon form of the augmented matrix without specifically writing down the solution sets (should they exist). (5 points each part)

a) $2x + 3y = 1$
$2x - 3y = 1$

b) $x - y = 1$
$2x - 2y = 2$

c) $x + y = 2$
$2x + 2y = 1$

2. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$. Determine conditions on the 3×1 matrix B such that the linear system $AX = B$ of 3 equations in 3 unknowns is inconsistent. (10 points).

3. Are the 2×2 matrices $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ inverses or not.

Why or why not ?

4) (a)Write down an example of a square matrix that is a zero divisor and show that it is a zero divisor. (5 points)

(b) Prove if A is a zero divisor then A is singular. (Hint: Use a proof by contradiction). (10 points)

5. Supposed that A is such that it can partitioned as $A = \begin{bmatrix} B & C \\ 0 & 0 \end{bmatrix}$ so that A is multiplicatively conformable with itself.

Prove that for all $n \geq 1$, $A^n = \begin{bmatrix} B^n & B^{n-1}C \\ 0 & 0 \end{bmatrix}$