MATH 207
Test I   Name________________________

I. Definitions & Statements of Theorems.

A. State the definition of the following terms.

1. Inverse of a matrix
2. Subspace of a vector space (definition not theorem)
3. Consistent system of equations
4. Rank of a matrix

B. Complete the statements of the following theorems.

1. If __________ then $Ax = b$ is a consistent system of equations
2. If A is __ $\times$__ and if $Ax = b$ is consistent, then $A_R =$_______

C. True or False. If False, then provide a "correction" to make the statement true.

1. There exists no inconsistent homogeneous system of equations.
2. Rank(A) $>$either m or n (or both).
3. An elementary matrix is invertible.
4. For an $m \times n$ matrix A, rank(A) $= n$ implies $Ax = b$ is consistent.
5. A consistent, nonhomogeneous system of equations always has infinitely many solutions.

II. Computations.

A. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$. Show the first couple of your steps and then use your calculator to finish the job.

B. Consider the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 1 & -1 \\ 2 & 6 & 4 & 2 & 1 \\ 3 & 8 & 3 & 3 & 0 \\ 1 & 4 & 5 & 1 & 2 \end{bmatrix}$

a. What is the row echelon form of A? (You need not show your (or your calculator's) work for this part.)

b. What is the rank of A?

c. How many solutions are there to $Ax = 0$?

d. Solve $Ax = 0$.

e. How many solutions are there to $Ax = b$ for $b = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -4 \end{bmatrix}$?

f. Solve $Ax = b$ from part (e).

C. Which of the following sets are vector spaces. You do not need to give a formal proof (on this test) of your answer, but convince me that you "know what's going on."

a. $S = \{f \mid f$ is continuous on $[0, 1]$ and $\lim_{x \to 1} f(x) = 0\}$.

b. S is the set of all 2×3 matrices such that the sum of the entries along the top row is not zero and the sum of the entries along the bottom row is zero.

III. Theorems. (In both statements, A is an $n \times n$ matrix.)

A. If A is nonsingular and k is a nonzero scalar, show that $k\,A^{-1}$ is nonsingular.

B. Prove: if A is singular, then $Ax = 0$ has infinitely many solutions.