MTH207. Fall 98. In-class Exam II.
Be Neat. Show work.

I. Give mathematically precise statements of the following: (4 pts. each)

1. The definition of subspace.
2. The Subspace Theorem.
3. The definition of Span $\{x_1, \ldots, x_n\}$.
4. The definition of Range T.
5. The Dimension Theorem.

II. Computational Problems (10 pts. each)

6. Let V be the real vector space of all real valued functions that are continuous on $[0,1]$. Let $W = \{f : f \in V \text{ and } \int_0^1 f(x)\,dx = 0\}$. Is W a subspace of V. Why?

7. Define $T : R_{1,2} \to R_2[x]$ by $T((a,b)) = (a+b)x + ab$. Is T linear? Why?

8. Define $T : R_3[x] \to R_{1,2}$ by $T(ax^2 + bx + c) = (a+b+c, -a-b)$. Note that T is linear. Find a basis for Range T. Is T one-to-one or not? Why?

9. Find a basis for $R_{1,3}$ that includes the vector $(1,2,0)$.

10. Let $T : R_{1,2} \to R_3[x]$ be the linear transformation defined by $T(a,b) = ax^2 + (a+b)x + (a-b)$. Let β be the standard ordered basis for $R_{1,2}$ and $\gamma = (x^2 + x, x, x-1)$ an ordered basis for $R_3[x]$. Find $\mathcal{R}_{\gamma\beta}(T)$.

11. Suppose you know that $T(x^2+1) = (1,2,0), T(x^2-1) = (0,1,2)$, $T(x) = (1,3,2)$ and that T is a linear transformation from $R_3[x]$ to $R_{1,3}$. Write down the general formula for T, if possible, or explain why it is not possible. Is T onto $R_{1,3}$? Why or why not?

III. More Theoretical Problems (10pts. each)

12. Recall that for sets A and B, $A \cap B = \{x : x \in A \text{ and } x \in B\}$. Suppose that A and B are finite subsets of the vector space V. Note that Span $A \cap$ Span B is a subspace of V. Prove that Span $A \cap B$ is a subspace of Span $A \cap$ Span B.

13. Let $V = \{f : f''(x) \text{ exists and is continuous}\}$. Let $W = \{g : g \text{ is a continuous function}\}$. V and W are vector spaces over R. Let $k \in R$. Define $T : V \to W$ by $T(f) = f''(x) + kf(x)$. Show T is linear. Show $f(x) = \sin x$ and $g(x) = \cos x$ are both elements in NullSpace T. Say as much as you can about NullSpace T based upon this fact. Justify your conclusion.