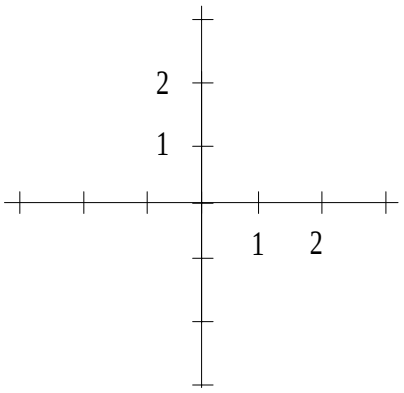
MATH 223
Test I Name _____

1.(25) Consider the function $z = f(x, y) = x - y^2$.

a) Sketch (and label) the level curves for $z = 0, 1, -1,$ and 2.



b) On the figure in (a) sketch the gradient $\nabla f(1, 2)$.

c) On the figure in (a) sketch $\nabla f(-2, 0)$.

d) On what level curve (i.e. $z =$??) is the point $(1, 2)$?

e) Calculate the projection of the vector $\vec{j}$ in the direction of $\nabla f(1, 2)$.

f) What is the directional derivative of f at the point $(1, 2)$ in the direction of $\vec{i} + \vec{j}$?

g) What is the value of the largest possible directional derivative of f at (1,2).

2.(10) Suppose that the price P (in dollars) to purchase a used car is a function of C, its original cost (in dollars) and its age A (in years). So $P = f(C, A)$.

a) What are the units of $\partial P/\partial A$? What would you expect for the sign of this partial derivative? (You need not explain why.)

b) What would you expect for the sign of the $\partial P/\partial C$? Explain why.

3.(15) Consider the function of three variables $w = h(x, y, z) = x^2 z + 2y\, e^{z-1}$.

a) What is an equation that gives the level surface of the function for $w = 5$?

b) Find a "generic" normal vector at an arbitrary point (x, y, z) on the surface in (a).

c) Given $x = 1$ and $z = 1$ what is the value of y so that the point $(1, y, 1)$ lies on the level surface in (a)?

d) Give a specific normal vector $\vec{n}$ at the point on the surface found in (c).

e) Find an equation for the tangent plane to the level surface in (a) at the point found in (c).

4.(25) Some computations

Let $z = f(x, y) = \sin(x^2 + y^2) + \frac{x}{y}$

a) $\partial z/\partial x =$

b) $f_y =$

c) $f_{xy} =$

d) $\partial^2 z/\partial x^2 =$

Now suppose that also we have $x = 3u + 2v$ and $y = u^2 v^2$
(Answer must be left in x, y, u, v)

e) $\partial z/\partial u =$

5.(10) Find an equation for the plane that passes through the point $(2, -1, 1)$ and contains the vectors $\vec{i} + 2\vec{j}$ and $\vec{i} - \vec{j} + 4\vec{k}$.

6.(5) The function $z = f(x, y) = \tan^{-1}(xy)$ is not simple enough for hand computations. We do know, however, that at $(.5,\ 2)$ the value of the function is $\pi/4$. (Right??) Use this fact and local linearity (i.e., a tangent plane) to approximate the value of $f(.6, 2.2)$. (Yes, yes, I know your calculator will give the "exact" value of $f(.6, 2.2)$ but that's not the point of the question.)

7.(10) Give the name of one of the surfaces and draw the other (your choice).

a) $x^2 + 4y^2 = 1$ b) $z = 1 - x^2 - y^2$