Math 223 Exam#2

Name ____________

I.D. # ____________

1. (32 pts) Given the function $f(x,y,z) = 3xz - 4x^2y + 5yz^2$,

(a) Find $f_x(x,y,z)$.

(b) Find $f_y(1,-1,2)$.

(c) Find $f_{zz}(x,y,z)$.

(d) Find $f_{yz}(1,-1,2)$.

(e) Find the gradient of $f(x,y,z)$.

(f) Find Find the gradient of $f(x,y,z)$ at $(1,-1,2)$.

(g) Find the directional derivative of $f(x,y,z)$ at $(1,-1,2)$ in the direction of $\overrightarrow{j} - 2\overrightarrow{k}$.

(h) Find the greatest increasing direction of $f(x, y, z)$ at (1,$-1, 2$).

2. (8 pts) Find an equation of the plane which tangents to the surface $2x - 3y^2 + 4xz = 7$ at $(1, -1, 2)$

3. (4 pts) Find the local linearization of the function $f(x, y) = x^3 e^y$ at $(2, 0)$.

4. (5 pts) Find the differential of the function $f(x, y) = x^2 y^3$ at $(1, 2)$

5. (6 pts) Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ of the function $z = \sqrt{x^2 + y^2}$ with $x = v\text{cos}u$ and $v\text{sin}u$.

6. (8 pts) Find the local maxima, local minima, and saddle points of the function $f(x, y) = x^3 - 3x + y^3 - 3y + 5$.

7. (9 pts) Determine if each of the following functions has global maxima and minima. (a) $f(x, y) = x^2 - 4x + y^4 - 8y^2 - 100$.

(b) $f(x, y) = x^2 + 8x - 3y + 100$.

(c) $f(x, y) = 5 - \sqrt{x^2 + y^2}$.

8. (8 pts) Find the least square regression line for the points $(1, -1)$, $(2, 1)$, and (3,2).

9. (8 pts) A box without top is going to be made to have volumn 32 cubic feet. Find the minimum square feet of materials required.

10. (12 pts) Use Lagrange multiplier to find the maximum and minimum values of $f(x, y) = 5 - x - y$ subject to $4x^2 + y^2 = 4$.