MATH 223

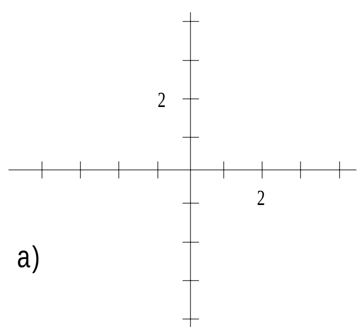
Test II Name ____________________

1.(10) Consider the table of values associated with a function $f(x, y)$. Each line of the table begins with the coordinates of a point and then gives the values of f and its derivatives. In the spaces at the right side of the table fill in properties of the function at the point such as "local max," "saddle point," "not a critical point," "cannot tell," etc.

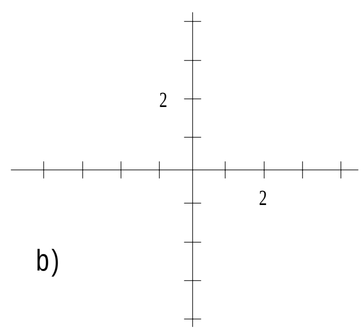
x	y	f	f_x	f_y	f_{xx}	f_{yy}	f_{xy}	properties of the function at the point
1	-2	3	0	0	2	1	1	
1	2	4	0	0	1	0	-1	
0	0	10	1	1	2	4	0	
2	3	0	0	0	-9	-1	2	
2	-2	1	0	0	0	2	0	

2.(10) Consider the function $f(x, y) = 2x + y$ and the constraint $x^2 + y^2 = 4$.

a) Sketch several level curves of f and the constraint. Label the "levels" of f on your sketch.



a)



b)

b) Sketch graphs of the levels that give possible extrema of f and mark with $(*)$ where those extrema occur. Tell how you can determine, from your figure, whether the extrema in your figure are maxima, minima, saddle points, or nothing at all?

c) Use the Lagrange multiplier method to find the extrema of f subject to the constraint.

3.(10) State the definition of **either** the double integral $\int_R f\,dA$ **or** the triple integral $\int_D f\,dV$. You may assume that R is a rectangle and D is a rectangular box. (This is to be the "official" definition of double or triple integral. Include enough details so that it is clear that you understand the definition.)

4.(10) Evaluate the integral. Show all steps. $\displaystyle\int_e^{e^2}\int_1^{\ln x}\frac{1}{x}\,dy\,dx$

5.(10) a) Sketch the region that is given by the limits of integration of the integral $\displaystyle\int_{-2}^{1}\int_{x^2}^{2-x} f\,dy\,dx.$

b) Reverse the order of integration.

6.(10) Sketch or describe carefully in words the region that is given by the limits of integration in the triple integral $\displaystyle\int_0^2\int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}}\int_{-1}^{5-x-y} f\;dz\,dx\,dy$

7.(10) Set up (do not evaluate) a double integral to find the volume of the solid bounded on the bottom by the paraboloid $z = x^2 + y^2$ and on the top by the plane $z = 4y$.

8.(10) Set up (do not evaluate) a triple integral to find the mass of an object in the first octant that is bounded by the coordinate planes, the vertical plane $y = 2 - 2x$, and the surface $z = 1 - x^2$, $0 \leq x \leq 1$. The density of the object at (x, y, z) is proportional to the distance from (x, y, z) to the origin.

9.(10) Convert the integral to either cylindrical or spherical coordinates (you need not evaluate the integral) $\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} \frac{1}{(x^2 + y^2 + z^2)^{1/2}} \, dz \, dy \, dx$

10.(10) Set up an integral to find the volume inside the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$.