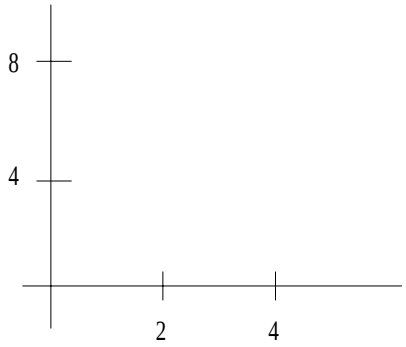
MATH 223
Test III NAME _____

1.(20) Let $\mathcal{R}$ denote the region described by the integral $\int_0^4 \int_x^{2x} x^2 + y^2 \, dy \, dx$.

a) Sketch the region $\mathcal{R}$. (note different scales)



b) Perform <u>one</u> integration on the integral. Leave your answer as a Calc I integral.

c) The integral $\int\int_{\mathcal{R}} f(x,y) \, dx \, dy$ (order reversed) is more work. Why?

d) Find appropriate limits to integrate with the order as in part (c).

2.(10) Consider the wedge shaped region described as follows. The sides are formed by the two vertical planes $x = y$ and $x = -y$ (for $x > 0$). The right side (when looked at from the first octant) is formed from the cylinder $x^2 + y^2 = 4$. The bottom is the xy-plane. And the top is the plane $z = x + y$. Assume the density at a point in the region is proportional to the distance from the point to the origin. SET UP a triple integral in cylindrical coordinates to find the mass of this solid.

3.(10) Find parametric equations of the line of intersection of the two planes $2x + y - z = 10$ and $3x - y + 4z = 5$. (For a point, try letting $z = 0$ and then add.)

4.(20) a) Describe in <u>words</u> the region that is described by the integral:

$$\int_0^3 \int_{-\sqrt{9-z^2}}^{\sqrt{9-z^2}} \int_{-\sqrt{9-y^2-z^2}}^{\sqrt{9-y^2-z^2}} x \, dx \, dy \, dz$$

(Be as specific as you can.)

b) Change the integral to either cylindrical to spherical coordinates.

c) Give parametric equations describing the curved surface that is the "top" of the region. (Be sure to give the limits on your parameters that describe the surface.)

d) Give a parametric representation of the boundary curve between the two different surfaces that form the boundary of the 3-dimensional region. Give limits on your parameter.

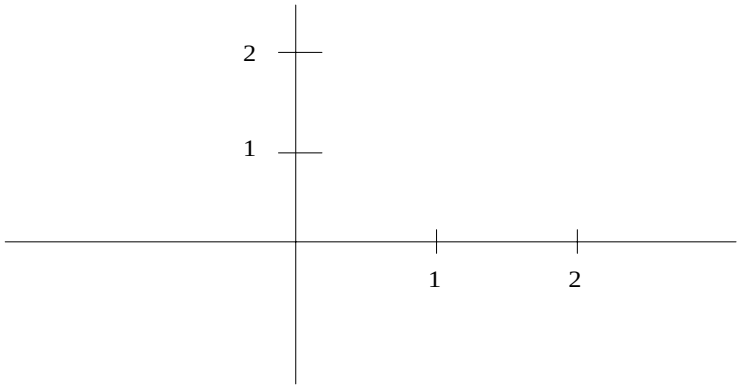
5.(10)The change of variables formula is $\int_R f(x,y) \, dA = \int_T f(x(s,t), y(s,t)) \, J(s,t) \, ds dt$

a) Tell a "well known" example of J.

b) What is J for the transformation $x = s^2 - 4t,\ y = 2s + 5t$.

c) What is the purpose of J? That is, why do we need to include it? (Answer in 2 or 3 full sentences.)

6.(20) a) Sketch the vector field $F(x, y) = 2x\,\vec{i} - y\,\vec{j}$



b) Calculate the flow for F.

c) Calculate the specific flow for F passing through $(1, 3)$ at time $t = 0$. Sketch it on your figure from (a).

d) Describe in words the flow that passes through $(2, 0)$. (Look at your figure in (a).)

7.(10) Suppose that the surface S is given by the equation

$$F(x, y, z) = xy + 3x^2z + z^3 = 9.$$

a) Tell how you know that the point $(2, -2, 1)$ lies on the surface.

b) Calculate $F_z(x, y, z)$.

c) What does the value of $F_z(2, -2, 1)$ tell you about the surface S near the point $(2, -2, 1)$? What theorem are you using here?