MTH223 Calculus III Exam III Fall 98. Name__________________

Be neat show work. All problems are worth the same number of points. (About 12.)

1. Let $f(x,y) = x^{1/3}y^{1/3}$ is continuous at (0, 0). It is not differentiable at $(0,0)$. Show that $f_x(0,0)$ and $f_y(0,0)$ exist then use the definition of differentiability at $(0,0)$ to show that it is not differentiable at $(0,0)$.

2. Find all local extrema and saddle points for $x^3 - 3x + y^3 - 3y$.

3. Find the maximum and minimum for $f(x,y) = x^2 + 2y^2$ when $x^2 + y^2 \leq 1$.

4. Evaluate $\int_0^1 \int_y^1 \sec^2(x^2)\, dx\, dy$.

5. Evaluate $\int_R e^{x^2+y^2}\, dA$ where R is the region in the circle of radius 2 in the 4th quadrant.

6. The density at a point P in a ball of radius 4 is proportional to the distance from P to the center of the ball. Write down an integral in rectangular coordinates that you can use to find the mass of the ball. Find the mass.

7. Given $\int_S f(x,y,z) dz\, dy\, dx$. Use the change of variables formula to convert to an integral in cylindrical coordinates over an appropriate region S'. Show work.

8. A system of point masses $P_1, \ldots, P_n$ are located at the points (x_k, y_k, z_k) and have masses m_k, for $k = 1, \ldots n$. The x-coordinate of the centroid of this system is given by

$$\overline{x} = \frac{\sum_{k=1}^{n} m_k x_k}{M}$$

where M is the total mass of the system. Assume a solid object of density $\delta(x,y,z)$ occupies the region S in 3-space. Assume δ is a continuous function. Derive the formula for $\overline{x}$, the x-coordinate of the centroid of the object. Hint: What do you need to do in order to be able to assume the density function is constant.