MATH 223
Exam 3

Name__________________________

1. Parametrize the circle of radius 3 centered at the origin and traced out clockwise.

2. A particle that passes through the point $P=(5,4,\,-2)$ at time $t=4$ is moving with constant velocity $\boldsymbol{v}=2\boldsymbol{i}-3\boldsymbol{j}+\boldsymbol{k}$. Find the parametric equations for its motion.

3. Use the Fundamental Theorem of Line Integrals to compute the line integral of the vector field $\boldsymbol{F}(x,y)=xy^2\boldsymbol{i}+x^2y\boldsymbol{j}$ along the curve given by

$$\boldsymbol{r}(u)=u\,sin\pi\,u\boldsymbol{i}+cos\pi\,u^2\boldsymbol{j},\;u\in[0,1].$$

4. Use Green's Theorem to evaluate $\int_c \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\mathbf{F}(x, y) = 3y\boldsymbol{i} + 5x\boldsymbol{j}$ and $C : x^2 + y^2 = 1$.

5. Use the Divergence Theorem to calculate the flux of the vector field

$$\boldsymbol{F}(x, y, z) = (x^2 + y^2)\boldsymbol{i} + (y^2 + z^2)\boldsymbol{j} + (x^2 + z^2)\boldsymbol{k}$$

through the cube given by $0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq z \leq 1.$

6. Use the Divergence Theorem to evaluate the flux integral

$$\int_S (x^2\boldsymbol{i} + (y - 2xy)\boldsymbol{j} + 10z\boldsymbol{k}) \cdot d\boldsymbol{A}$$

where S is the sphere of radius 5 centered at the origin, oriented outward.

7. Compute the line integral of the vector field $\boldsymbol{F}(x, y) = x^2\boldsymbol{i} + y^2\boldsymbol{j}$ along the line from the point $(1, 2)$ to the point $(3, 4)$.

8. Compute the flux of the vector field $\boldsymbol{F}$ through the surface S where

$$\boldsymbol{F}(x, y, z) = 2x\boldsymbol{j} + y\boldsymbol{k}$$

and S is the part of the surface $z = -y + 1$ above the square $0 \leq x \leq 1,\ 0 \leq y \leq 1$ oriented upward.

E. Use Green's Theorem to evaluate $\int_c \boldsymbol{F} \cdot d\boldsymbol{r}$ where

$$\mathbf{F}(x, y) = (3xy + y^2)\boldsymbol{i} + (2xy + 5x^2)\boldsymbol{j}$$

and $C : (x-1)^2 + (y+2)^2 = 1$.