MTH 223 Final Exam December 17, 1997

Part I True/False Questions

If the statement is **FALSE** state your reason(s). (3 points each)

1._____T/F If $w = f(x, y, z) = 4 - \cos(x^2 + y^2 + z^2)$, there is a level surface for every value of w.

2._____T/F The curve represented by the position vector $\vec{r} = \ < 1, y, 2 >$ is a line perpendicular to the xz-plane.

3._____ T/F Given $z = x^2\cos^2(y) + y^2\sin^2(y)$. Since $\sin^2(y) = 1 - \cos^2(y)$, $z_{xy} + z_{yx} = 0$.

4._____ T/F If $f(x, y)$ has a saddle point at (a, b), then $\nabla f(a, b) = \ < 0, 0 >$.

5.______ T/F A double integral of the form $\int_a^b \int_c^d 1\, dp\, dq$, a,b,c and d are any constants, is always greater than zero.

6.______ T/F In 3D space, the position vector $\vec{r}(t) = 5\vec{i} + 5\cos(t)\vec{j} + 5\sin(t)\vec{k}$ represents a circle in the yz-plane.

7.______ T/F The graph of a flow of a vector field is perpendicular to the graph of the vector field itself.

8._____ T/F If a line integral is evaluated using two different parameterizations of the same curve with the same orientation, the values of the line integrals will be identical.

9._____ T/F For a constant vector field, any flux integral through a rectangular planar surface can be computed without using an integral.

10.____ T/F Gauss' divergence theorem is only true on Wednesdays!

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

Part II The Real Problems!

1.[10] a) Two vectors $\vec{v}$ and $\vec{w}$ are said to be orthogonal or perpendicular if ....

b) Find a and b so that $< a, b, 2 >$ is orthogonal to both $< 2, -1, 3 >$ and $< -4, 1, 5 >$.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

2.[15] Given the vectors $\vec{a} = \; < 1, -1, 2 >$ and $\vec{b} = \; < 0, 1, -3 >$.

a) Find a unit normal vector for all planes which are parallel to the plane containing the given vectors.

b) Find an equation of a plane through (5,6,−4) which is parallel to the plane containing the given vectors.

c) Determine the vector equation of a line through $(2, 4, -5)$ which is perpendicular to the plane of part b).

3.[15] Let $f(x, y) = x^2 e^{3y}$.

a) Find the linear function that gives a good local approximation to f near $(1, 0)$.

b) Evaluate: $f(2, 1)$ and your estimate from (a) at $(2, 1)$. Give a geometric reason why the estimate is smaller than the value of the function.

c) Find a unit vector that points in the direction of greatest change of the function f at the point $(1, 0)$.

4.[10] A rectangular parallelepiped in the first octant has three of its faces in the coordinate planes and its vertex opposite the origin is in the first octant and on the plane 2x + y + 3z = 6. Find the maximum volume of the rectangular parallelepiped. You may use Lagrange's method if you wish.

5.[15] Consider the region in the first octant described as follows:

i) inside the vertical cylinder $x^2 + y^2 = 4$

ii) below the plane $x + y + 4z = 4$

a) Sketch the region

b) SET UP a double integral using x and y to find the volume

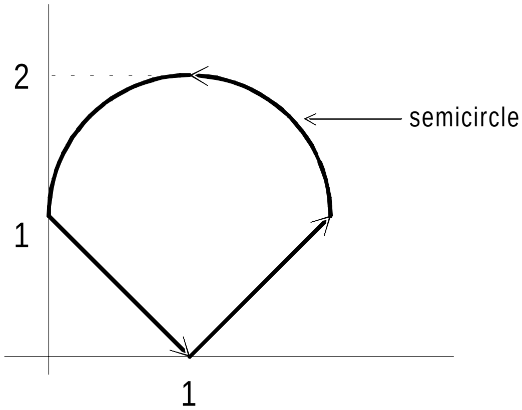
c) SET UP a double integral using r and θ to find the volume

d) Compute the volume. Show all of your work. Check with the TI-92

e) Is your answer reasonable? Explain why or why not.

6.[10] Let $\vec{F} = (x^2 - 2y)\vec{i} + (\cos(y) + 3x)\vec{j}$

a) Calculating the line integral $\int_C \vec{F} \bullet d\vec{r}$ for the curve shown in the figure would involve *considerable* computation. Why is this computation complicated? (Do not attempt to actually compute the line integral.)



b) Use Green's Theorem to compute the line integral.

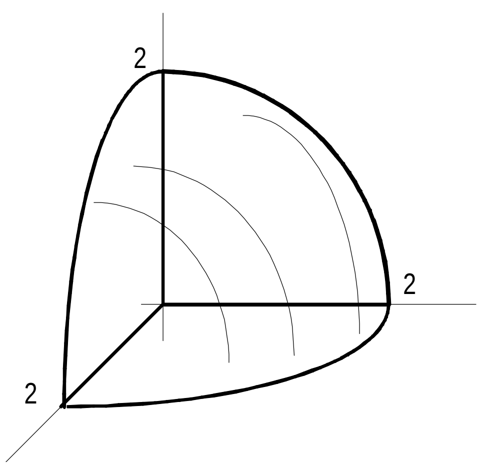
7.[10] Let C be the curve <u>clockwise</u> on the unit circle from (0, 1) to (1, 0).

a) Determine parametric equations for C. (Be sure to give the relevant interval for the parameter.)

b) Evaluate the line integral $\int_C \vec{F} \bullet d\vec{r}$, where

$\vec{F} = (e^y + 2xy)\vec{i} + (xe^y + x^2 - 3y^2)\vec{j}$ [Hint: Is $\vec{F}$ a gradient field?]

8.[15] Consider the region shown in the figure. The curved part is one-eighth of the sphere of radius 2 centered at the origin. The other three sides are the coordinate planes.



Let $\vec{F} = (3x - y^2)\vec{i} + 2y\,\vec{j} + (e^{x+y} - z)\vec{k}$

a) Compute the flux out of the back surface (quarter circle in the yz-plane)

b) Use a theorem to find the total flux out of the entire region.