MATH 223 Final Exam Spring 1999

Name______________________________

1. Find the equation of the tangent plane at the given point.

$$z = e^y + x + x^2 + 6 \text{ at the point } (1, 0, 9)$$

2. Find the local maxima, local minima, and saddle points of the given function.

$$f(x, y) = (x + y)(xy + 1)$$

3. Use Lagrange multipliers to find the maximum and minimum values.

$$f(x, y, z) = x^2 - 2y + 2z^2,\ x^2 + y^2 + z^2 = 1$$

4. Evaluate $\int_R sin(x^2 + y^2)dA$ where R is the disk of radius 2 centered at the origin.

5. Find the velocity vector $\boldsymbol{v}(t)$ for the given motion of the particle. Also find the speed $||\boldsymbol{v}(t)||$ and any times when the particle comes to a stop.

$$x = t^2 - 2t,\ y = t^3,\ z = 3t^4 - 4t^3$$

6. Compute the flux of the vector field $\boldsymbol{F}$ through the given surface S.

$\boldsymbol{F} = z\boldsymbol{i} + y\boldsymbol{j} + 2x\boldsymbol{k}$ and S is the rectangle $z = 4, 0 \leq x \leq 2, 0 \leq y \leq 3$ oriented in the positive z-direction.

7. Compute the line integral of the vector field along the given path.

$\boldsymbol{F} = x^3\boldsymbol{i} + y^2\boldsymbol{j} + z\boldsymbol{k}$ and C is the line from the origin to the point $(2, 3, 4)$

8. Compute the line integral of the vector field along the given path.

$\boldsymbol{F}(x, y) = x^2\boldsymbol{i} + y^2\boldsymbol{j}$ and C is the line from the point $(1, 2)$ to the point $(3, 4)$

9. Use Green's Theorem to evaluate the line integral along the positively oriented curve.

$\int_C (y^2 - tan^{-1}x)dx + (3x + siny)dy,$ C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 4$

10. Use the Divergence Theorem to calculate the surface integral $\int\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$ where $\boldsymbol{F}(x, y, z) = ye^{z^2}\boldsymbol{i} + y^2\boldsymbol{j} + e^{xy}\boldsymbol{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = y + 3$.