

Math 325 Exam#3

Name _____

ID# _____

1. (4 pts) Suppose that random variable Y possesses the density function

$$f(y) = \begin{cases} cy^2, & 0 \leq y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the value of c .

2. (10 pts) Suppose that random variable Y possesses the density function

$$f(y) = \begin{cases} \frac{3}{4}y(2-y), & 0 \leq y \leq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the cumulative function $F(y)$.
(b) Find $P(Y > 1)$.

3. (10 pts) Suppose that random variable Y possesses the cumulative function

$$F(y) = \begin{cases} 0, & y < 0 \\ \frac{1}{4}y^2, & 0 \leq y \leq 2 \\ 1, & y > 2. \end{cases}$$

- (a) Find the density function $f(y)$.
(b) Find $P(Y > 1)$.

4. (16 pts) Weekly CPU time used by an accounting firm has a density function (measured in hours) given by

$$f(y) = \begin{cases} \frac{3}{4}y(2-y), & 0 \leq y \leq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the mean and variance of weekly CPU time.
(b) The CPU time cost the firm \$150 per hour. Find the mean and variance of weekly cost for CPU time.

5. (20 pts) Let Z be the standard normal random variable.

(a) Find $P(0 < Z < 1.25)$;

(b) Find $P(-1 < Z < 1.25)$;

(c) Find $P(1 < Z < 1.25)$;

(d) Find the value z_0 such that $P(Z > z_0) = 0.01$;

(e) Find the value z_0 such that $P(-z_0 < Z < z_0) = 0.95$.

6. (16 pts) Scores on Scholastic Aptitude Test (SAT) for verbal ability of high school seniors are normal distributed with mean 430 and standard deviation 100.

(a) What percent of scores are between 300 and 500?

(b) How high must a student score in order to place in the top 10% of all students taking the SAT?

7. (10 pts) Suppose that the time between incoming calls to a telephone switchboard is exponential distributed with a mean of 30 seconds.

(a) What is the probability that there will be no calls during a 1-minute period?

(b) If an operator wants to go to the coffee room to get a cup of coffee, but only wants to take 50% chance to miss a call, how soon must she come back?

8. (5 pts) A random variable Y has the density function

$$f(y) = \begin{cases} e^{-y}, & 0 \leq y < \infty \\ 0, & \text{elsewhere.} \end{cases}$$

Find the moment-generating function for Y .

9. A random variable Y has a chi-square distribution with $\nu = 8$ degrees of freedom.

(a) (3 pts) What is the mean, variance, and standard deviation of Y ?

(b) (3 pts) Use Tchebysheff's inequality to find an interval for which the probability Y will lie within it is at least .75.

(c) (3 pts) Use Tchebysheff's inequality to find an upper bound for the probability $P(Y > 30)$.