Math 325 Exam#4

Name _____

ID# _____

1. (6 pts) Suppose that random variable Y_1 and Y_2 have the density function

$$f(y_1, y_2) = \begin{cases} c(1 - y_1^2), & 0 \le y_2 \le y_1 \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the value of c.

2. (30 pts) Suppose that random variable Y_1 and Y_2 have the density function

$$f(y_1, y_2) = \begin{cases} 24y_1(1 - y_2), & 0 \le y_1 \le y_2 \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the density function of Y_2.

(b) Find $P(Y_2 > 0.5)$.

(c) Find the density function of Y_1 given $Y_2 = 0.5$.

(d) Find $P(Y_1 > 0.25 | Y_2 = 0.5)$.

(e) Find $P(Y_1 > 0.25 | Y_2 \le 0.5)$.

3. (24 pts) Suppose that random variable Y_1 and Y_2 have the density function

$$f(y_1, y_2) = \begin{cases} 24y_1y_2, & y_1 \geq 0, y_2 \geq 0, \text{ and } y_1 + y_2 \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $E(Y_1)$.

(b) Find $V(Y_1)$.

(c) Find $Cov(Y_1, Y_2)$.

(d) Are Y_1 and Y_2 independent?

4. (12 pts) Consider a sample of size 2 drawn without replacement from an urn containing three balls, numbered 1, 2, and 3. Let Y_1 be the smaller of the two numbers drawn and Y_2 the larger.

(a) Find the joint discrete density function of Y_1 and Y_2.

(b) Find $P(Y_1 = 1|Y_2 = 3)$.

5. (12 pts) A firm purchases two types of industrial chemicals. The type I chemical costs \$5.00 per gallon, whereas type II chemical costs \$4.00 per gallon. The mean and variance for the number of gallons of type I chemical purchased per week, Y_1, are 30 and 3, respectively. The amount of type II chemical purchased per week, Y_2, has mean 55 gallons and variance 6 gallons. If the covariance between Y_1 and Y_2 is $Cov(Y_1, Y_2) = -2$, find the mean and variance of the total amount of money spent per week on the two chemicals.

6. (14 pts) Consider a quality control plan that calls for randomly selecting 20 items from the daily production (assumed large) of a certain machine and observing the number of defectives. The proportion p of defectives produced by the machine varies from day to day and has a uniform distrbution on the interval (0, 1/2).
   (a) Find the expected number of defectives oberved among the 20 sampled items.

   (b) Find the variance of the number of defectives among the 20 sampled items.