Math 325 Final

Name ________________
ID# ________________

1. (10 pts) Companies A and B produce the same kind of parts. Company A produces 35% of the parts and 8% of them are defective. Company B produces 65% of the parts and 6% of them are defective. One part is randonly selected.
   (a) What is the probability that this part is defective?

   (b) If the part is tested to be defective, what is the probability that this part is produced by company A?

2. (10 pts) Among 10 parts, 2 are defective. A sample of 3 parts is selected for testing from these 10 parts. What is the probability that
   (a) None of them will be defective?
   (b) At least one of them will be defective?

3. (10 pts) A company produces 5% defective parts. Ten parts are randomly selected. What is the probability that
   (a) Exactly one of them will be defective?
   (b) At least one of them will be defective?

4. (10 pts) Let X be a random variable distributed as shown in the table below.

X	1	2	3	4
prob	.1	.2	.3	.4

Determine $E(X)$ and $V(X)$.

5. (10 pts) It is known that the number of incoming airplanes per minute at a large metropolitan airport is a random variable having a Poisson distribution with mean $\lambda = 1.5$.
   (a) Find the probability that there will be more than 10 incoming airplanes during a period of 5 minutes.

   (b) The time between incoming airplanes is exponential distrbuted with mean $\beta = \frac{2}{3}$. One airplane is just arrived. What is the probability that there will be no airplanes arriving in 2 minutes.

6. (10 pts) Suppose you are playing a game by paying \$5 up front, then you toss a balanced coin until a tail occurs. You will win \$2 on each occurrence of a head. Find the mean and variance of your net gain in ten games.

7. (14 pts) A and B play the following game: A tosses 3 fair coins and B tosses 4 fair coins. The player throwing the greater number of heads wins. In case of a tie, the throws are repeated until a winner is determined.
   (a) What is the probability that A wins on the first play?
   (b) What is the probability that A wins the game?

8. (20 pts) Suppose that random variable X has the density function

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find the cumulative function of X.
   (b) Find the mean value of X.
   (c) Find the variance of X.
   (d) What is the probability that exactly two of three independent observations on X are greater the $\frac{1}{2}$.

9. (10 pts) Suppose that random variable X has the density function

$$f(x) = \begin{cases} |x|, & -1 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the density function of $Y = 2X + 1$.

10. (15 pts) Suppose that X is a normal random variable with mean $\mu = 12$ and variance $\sigma^2 = 9$.
(a) Find $P(X > 10)$;

(b) Find $P(10 < X < 15)$;

(c) Find the value x_0 such that $P(X > x_0) = 0.01$;

11. (10 pts) Let Y_1, Y_2, and Y_3 be independent, normal random variables, each with mean $\mu = 3$ and variance $\sigma^2 = 4$. Find the density function of $X = Y_1 + 2Y_2 + 3Y_3$.

12. (10 pts) Let Y_1, Y_2, and Y_3 be independent, exponentially distributed random variables with mean $\beta = 2$. Find the density function of $Y = min\{Y_1, Y_2, Y_3\}$.

13. (16 pts) A bottling company uses a filling machine to fill plastic bottles with a popular cola with mean $\mu = 300$ ml and standard deviation $\sigma = 2.5$ ml. A sample of 36 bottles is taken.
    (a) What is the probability that the average contents $\bar{X}$ will be less than 299 ml?
    (b) How large should the sample size be in order to have the probability $P(\bar{X} < 299)$ less than .001.

14. Suppose that random variable X has the density function

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

    (a) (10 pts) Find the moment generating function of X.
    (b) (5 pts) Find the 20th moment of X.

15. (10 pts) If a random variable Y has moment-generating function $M(t) = \frac{e^t}{3-2e^t}$, find the probability that $Y = 3$.

16. (20 pts) To estimate the average miles per gallon (MPG) of certain type of cars, a sample of 6 cars is taken. The sample mean is $\bar{X} = 26$ and the sample standard deviation is $S = 2.3664$. Assume that MPG of a car is normal distributed.
    (a) Construct a 99% confidence interval for the average MPG of this type of cars
    (b) Test the null hypothesis H_O: $\mu = 29$ against the alternative hypothesis H_a: $\mu < 29$ at the significant level $\alpha = 0.05$.